Supply Identification for Water Distribution Systems

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Identifying the precise source of delivered flow in water distribution systems is increasingly important. The necessity for this arises when individual supplies exceed allowable levels of certain chemical or biological constituents. An explicit methodology to make these calculations for complex water distribution systems is presented. The methodology requires the solution of a set of linear equations of order equal to the number of junctions. The equation coefficients are based on the flow distribution for the system demands under consideration. Because the explicit solution procedure requires the simultaneous solution of large equation sets for large networks, an alternative solution technique was developed. This technique solved the equations in a simple, cyclic manner requiring little computer storage and proved to be an efficient scheme for carrying out the calculations.

During the past few decades, various computer models have been developed for use in modeling water distribution systems. These models can be used to predict flows and pressures throughout a particular distribution system in response to a given set of operating conditions. In systems with multiple water supply sources, it may be necessary to determine the contribution of each source to the demand at a particular junction node. Such calculations are normally required because of concern over the possible effects of one or more supplies that may exceed allowable levels of various pollutants or chemicals. As a result, it may be necessary to determine the percentage of flow originating at such supplies at critical points throughout the distribution system.

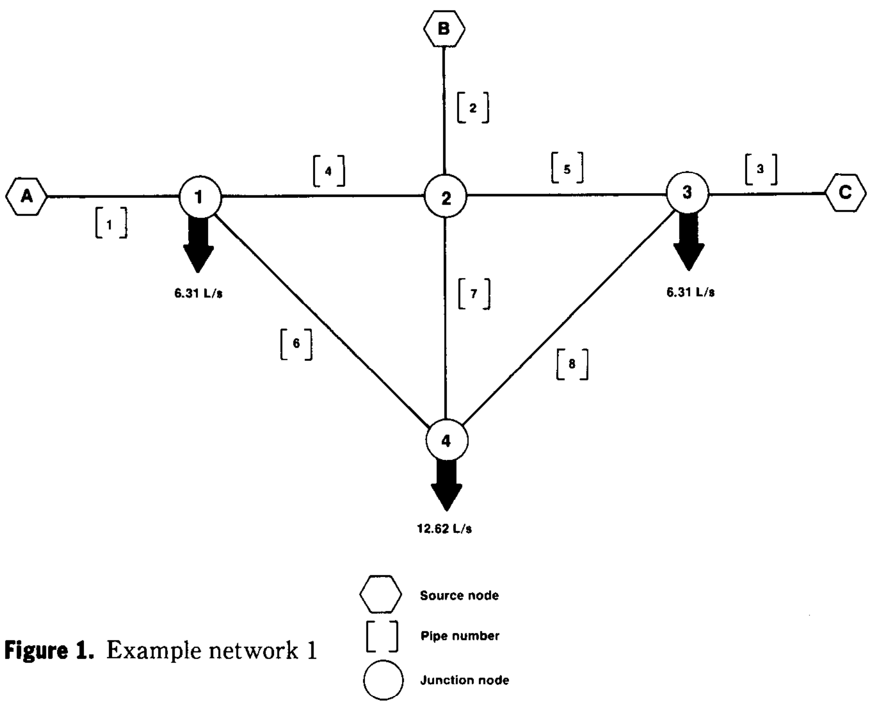

Figure 1. Example network 1

A methodology for solving this problem was first suggested by Wood in 1980 in an article dealing with slurry flow in pipe networks.[1] The procedure for calculating solids concentrations throughout the network is applicable to water distribution systems. More recently Clark et al[2] and Males et al[3] used a similar approach to solve the problem of mixing water from various sources in a pipe network. They also addressed the problem of determining the travel time from any source in the network to a point in the distribution system, as well as the problem of determining the cost of delivered water at any node.[4] In each case, a set of simultaneous equations was solved using a sparse-matrix-solution code.

Two simplified methods for determining nodal source contributions are examined. The two methods include the use of perturbation analysis and an iterative cyclic procedure. Although perturbation analysis is shown to be unsatisfactory for this type of analysis, the proposed iterative procedure is shown to be a fast and efficient method for solving the resulting system of equations.

Analysis

For a given distribution system, it is possible to define a contribution factor, $C(i,j)$, for a particular supply (i) that applies to all the flow out of junction node (j). The contribution factor is defined as the percentage of the outflow from the junction originating at the applicable supply i, and is based on the concept that all flow entering a node is mixed so that all outflow has a single contribution factor, which is determined from the contribution of the inflows to that node. To completely describe the distribution system, it is necessary to determine the contribution factor for each junction node in the system.

The procedure for determining the contribution factor vector is formulated as follows. For each i and each j the following relation applies:

$$\sum_{n \in [j]}^{N_j} C(i,n)\,Q(n) - C(i,j)\,\Sigma\,Q_o(j) = -\,Q_s(i) \quad (1)$$

in which N_j is the number of adjacent nodes with flow into node j and $\{j\}$ is the set of adjacent nodes with flow into node j. $C(i,n)$ are the contribution factors for the adjacent nodes and $Q(n)$ is the flow from the adjacent nodes to node j. $C(i,j)$ is the contribution factor determined for node j and $\Sigma Q_o(j)$ is the total outflow for node j. This term includes all flow leaving in pipes going to adjacent junction nodes and to fixed-grade nodes (such as tanks) and all external demand leaving the distribution system at node j. $Q_s(i)$ is the flow from supply i to node j and is zero unless this supply is directly connected to node j.

Equation 1 applies to each junction node in the system, so for a system of N_j junction nodes, a set of N_j simultaneous equations can be developed to determine the contribution factors for each of the N_j junction nodes. In matrix notation, this system may be expressed as

$$A\tilde{x} = b \quad (2)$$

in which the coefficient matrix A contains the coefficients of the contribution factors [i.e., $Q(n)$ and $\Sigma Q_o(j)$], the decision vector $\tilde{x}$ is made up of the unknown contribution factors, and the right-hand-side vector b contains the set of flows from the supply nodes [i.e., $Q_s(i)$]. Because the equation set is linear, conventional matrix techniques can be utilized to determine the solution.

Illustration

This procedure is applied to the example network shown in Figure 1. All junction nodes have a ground elevation of 30.5 m and all source nodes have a hydraulic grade of 61 m. The remaining physical characteristics of the network are given in Table 1. For the operating conditions illustrated in Figure 1, the hydraulic results are given in Table 2. The contribution factors for all junction nodes may be obtained by solving Eq 2 for each individual source. For the example network, the coefficient matrix A may be expressed as follows:

$$\begin{bmatrix} -\Sigma Q_o(1) & Q(3) & Q(3) & Q(4) \\ Q(1) & -\Sigma Q_o(2) & Q(3) & Q(4) \\ Q(1) & Q(2) & -\Sigma Q_o(3) & Q(4) \\ Q(1) & Q(2) & Q(3) & -\Sigma Q_o(4) \end{bmatrix} \quad (3)$$

For source A (Figure 1), the decision vector $\tilde{x}$ may be expressed as

$$\tilde{x} = \begin{bmatrix} C(\text{A},1) \\ C(\text{A},2) \\ C(\text{A},2) \\ C(\text{A},4) \end{bmatrix} \quad (4)$$

TABLE 1
Physical characteristics of example network 1

Pipe Number	Length *m*	Diameter *cm*	Roughness
1	304.8	30.5	100.0
2	304.8	25.4	100.0
3	304.8	20.3	100.0
4	304.8	25.4	100.0
5	304.8	25.4	100.0
6	304.8	20.3	100.0
7	304.8	15.2	100.0
8	304.8	20.3	100.0

TABLE 2
Hydraulic results for example network 1

Parameter	Nodes	Node Number	Flow Rate *L/s*	Head Loss *m*	Demand *L/s*	Grade Line *m*	Pressure *kPa*
Pipe number							
1	0	1	12.28	0.06			
2	0	2	7.62	0.06			
3	0	3	5.34	0.09			
4	1	2	0.42	0.00			
5	2	3	5.44	0.03			
6	1	4	5.55	0.10			
7	2	4	2.60	0.10			
8	3	4	4.47	0.07			
Junction number							
1					6.31	60.90	298.32
2					0.00	60.90	298.32
3					6.31	60.87	298.00
4					12.62	60.80	297.34

TABLE 3
Contribution factors for example network 1

Node	Source A	Source B	Source C
1	1.000	0.000	0.000
2	0.052	0.948	0.000
3	0.026	0.478	0.496
4	0.460	0.365	0.175

TABLE 4
Contribution factors for junction node 4

Source	ΔQ—L/s	C(i,4)	C*(i,4)	Error percent
A	0.62	0.490	0.460	7
B	0.38	0.305	0.365	16
C	0.26	0.205	0.176	16
Total ΔD	1.26			

TABLE 5

*Physical characteristics of example network 3**

Parameter	Node 1	Node 2	Length *ft*	Diameter *in.*	HW-C Value	Sum-M Factor	Pump Type	FGN Grade *ft*	Demand *gpm*	Elevation *ft*
Pipe number										
1	1	2	600.0	10.0	100.0	0.0	0.0			
2	0	2	500.0	10.0	100.0	0.0	1.0	800.0		
3	2	3	600.0	10.0	100.0	0.0	0.0			
4	3	4	1,900.0	10.0	100.0	0.0	0.0			
5	4	5	4,200.0	10.0	100.0	0.0	0.0			
6	5	6	400.0	10.0	100.0	0.0	0.0			
7	6	7	750.0	10.0	100.0	0.0	0.0			
8	7	8	700.0	6.0	100.0	0.0	0.0			
9	8	9	2,000.0	6.0	100.0	0.0	0.0			
10	9	10	2,000.0	6.0	100.0	0.0	0.0			
11	10	11	3,000.0	6.0	100.0	0.0	0.0			
12	10	12	2,300.0	10.0	100.0	0.0	0.0			
13	12	13	1,700.0	10.0	100.0	0.0	0.0			
14	13	14	1,700.0	10.0	100.0	0.0	0.0			
15	14	15	1,800.0	10.0	100.0	0.0	0.0			
16	0	15	1,100.0	16.0	100.0	0.0	1.0	795.0		
17	15	16	1,800.0	12.0	100.0	0.0	0.0			
18	16	17	2,400.0	12.0	100.0	0.0	0.0			
19	17	18	4,300.0	12.0	100.0	0.0	0.0			
20	18	19	2,300.0	10.0	100.0	0.0	0.0			
21	19	3	1,400.0	10.0	100.0	0.0	0.0			
22	4	20	2,200.0	6.0	100.0	0.0	0.0			
23	19	20	3,000.0	10.0	100.0	0.0	0.0			
24	20	28	3,400.0	6.0	100.0	0.0	0.0			
25	20	32	2,500.0	10.0	100.0	0.0	0.0			
26	32	17	1,200.0	10.0	100.0	0.0	0.0			
27	32	33	2,500.0	6.0	100.0	0.0	0.0			
28	33	14	2,200.0	6.0	100.0	0.0	0.0			
29	33	16	1,500.0	10.0	100.0	0.0	0.0			
30	28	33	2,400.0	10.0	100.0	0.0	0.0			
31	30	14	2,600.0	10.0	100.0	0.0	0.0			
32	31	13	1,400.0	6.0	100.0	0.0	0.0			
33	10	31	2,000.0	6.0	100.0	0.0	0.0			
34	30	31	1,500.0	6.0	100.0	0.0	0.0			
35	21	28	1,500.0	6.0	100.0	0.0	0.0			
36	25	28	750.0	6.0	100.0	0.0	0.0			
37	28	29	750.0	6.0	100.0	0.0	0.0			
38	26	29	750.0	6.0	100.0	0.0	0.0			
39	29	30	750.0	6.0	100.0	0.0	0.0			
40	27	30	750.0	6.0	100.0	0.0	0.0			
41	26	27	750.0	6.0	100.0	0.0	0.0			
42	25	26	750.0	6.0	100.0	0.0	0.0			
43	22	25	750.0	6.0	100.0	0.0	0.0			
44	23	26	750.0	6.0	100.0	0.0	0.0			
45	24	27	750.0	6.0	100.0	0.0	0.0			
46	24	23	750.0	6.0	100.0	0.0	0.0			
47	22	23	750.0	6.0	100.0	0.0	0.0			
48	21	22	500.0	6.0	100.0	0.0	0.0			
49	5	21	700.0	6.0	100.0	0.0	0.0			
50	7	22	1,600.0	6.0	100.0	0.0	0.0			
51	9	24	1,200.0	6.0	100.0	0.0	0.0			
52	0	8	500.0	6.0	100.0	0.0	1.0	800.0		
53	0	11	200.0	6.0	120.0	0.0	1.0	805.0		
Junction number										
1									150.0	969.0
2									0.0	970.0
3									100.0	972.0
4									100.0	974.0
5									100.0	985.0
6									100.0	977.0
7									100.0	980.0
8									0.0	982.0
9									100.0	975.0
10									100.0	948.0
11									100.0	964.0
12									100.0	932.0
13									100.0	923.0
14									100.0	919.0
15									0.0	903.0
16									100.0	920.0
17									0.0	948.0
18									100.0	962.0
19									0.0	972.0
20									100.0	972.0
21									150.0	978.0
22									150.0	975.0
23									150.0	968.0
24									150.0	965.0
25									150.0	969.0
26									150.0	965.0
27									150.0	962.0
28									150.0	966.0
29									150.0	962.0
30									150.0	954.0
31									100.0	937.0
32									100.0	958.0
33									100.0	937.0

*Metric conversions—3.28 ft = 1 m, 39.37 in. = 1 m, 15,839 gpm = 1 m^3/s

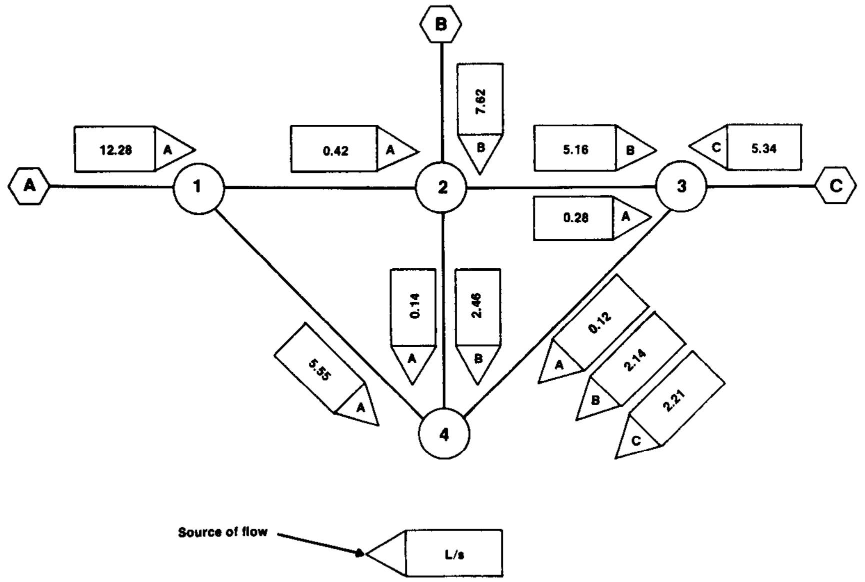

Figure 2. Flow distribution for example network

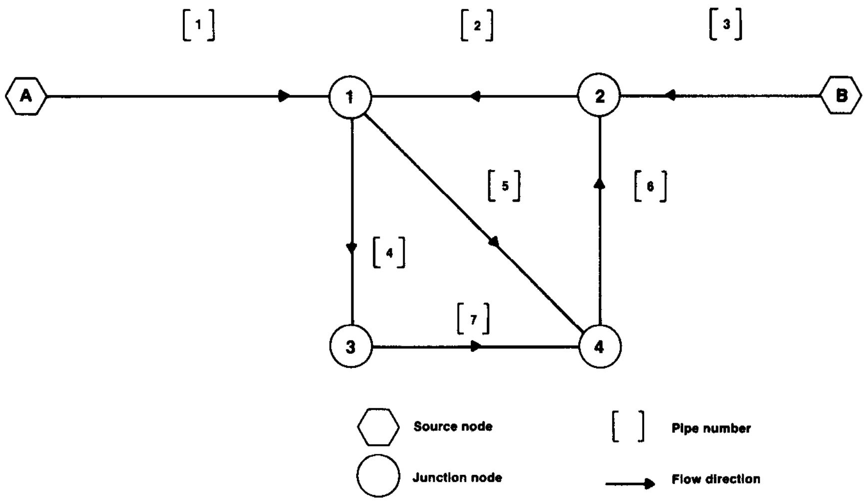

Figure 3. Example network 2

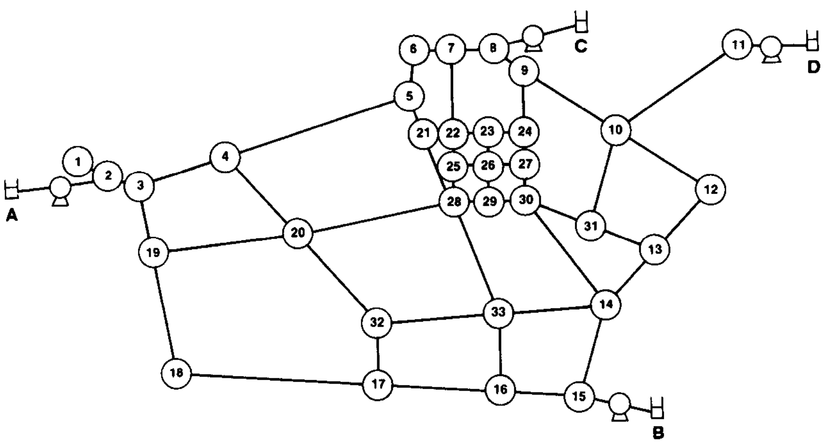

Figure 4. Example network 3

and the right-hand-side vector b may be expressed as

$$b = \begin{bmatrix} -Q_s(\text{A}) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

Substitution of the network hydraulic results that are given in Table 2 into Eqs 3 and 5 yields the following system of equations:

$$\begin{bmatrix} -12.28 & 0.00 & 0.00 & 0.00 \\ 0.42 & -8.04 & 0.00 & 0.00 \\ 0.00 & 5.44 & -10.78 & 0.00 \\ 5.55 & 2.60 & 4.47 & -12.62 \end{bmatrix} \begin{bmatrix} C(\text{A},1) \\ C(\text{A},2) \\ C(\text{A},3) \\ C(\text{A},4) \end{bmatrix} = \begin{bmatrix} -12.28 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix} \tag{6}$$

Solution of Eq 6 for the unknown contribution factors yields the values C (A, 1) = 1.000; C(A, 2) = 0.052; C(A, 3) = 0.026; and C(A, 4) = 0.460. This means that 100.0 percent of the demand at junction 1 is supplied from source A, 5.2 percent of the demand at junction 2 is supplied from source A, 2.6 percent of the demand at junction 3 is supplied from source A, and 46.0 percent of the demand at junction 4 is supplied from source A.

Only a slight modification of the equation set is required to consider other supplies. The term $Q_s(i)$ varies for each supply, and because this term appears in only one equation, the equation set is modified only slightly for each supply considered. Also several supplies can be considered simultaneously by including the appropriate $Q_s(i)$ terms. If all supplies in the distribution system are considered, then the contribution factors for any node add up to one. This is

$$\sum_{i=1}^{N_s} C(i,j) = 1 \tag{7}$$

in which N_s is the number of supplies for the distribution system. The contribution factors for all junction nodes for the example network are summarized in Table 3.

Alternative solution methodologies

The previously described procedure requires the simultaneous solution of a set of equations of order equaling the number of junction nodes in the pipe network. For large networks, this would require the solution of large equation sets. Unless special techniques are utilized for handling large arrays, the capacity of most standard microcomputers will be reached for systems greater than 150 nodes. In an attempt to develop a less cumbersome solution methodology, two alternative approaches were considered.

TABLE 6

*Hydraulic results for example network 3**

Parameter	Node 1	Node 2	Flow Rate *gpm*	Head Loss	Minor Loss	Pump Head	Line Velocity	HL 1,000	Elevation *ft*	Demand *gpm*	Pressure *psi*	Hydraulic Grade *ft*
Pipe number												
1	1	2	-150.00	0.18	0.00	0.00	0.61	0.30				
2	0	2	1,089.26	5.87	0.00	284.28	4.45	11.74				
3	2	3	939.26	5.35	0.00	0.00	3.84	8.92				
4	3	4	428.78	3.97	0.00	0.00	1.75	2.09				
5	4	5	296.90	4.44	0.00	0.00	1.21	1.06				
6	5	6	-16.87	0.00	0.00	0.00	0.07	0.01				
7	6	7	-116.87	0.14	0.00	0.00	0.48	0.19				
8	7	8	-376.14	13.80	0.00	0.00	4.27	19.71				
9	8	9	225.92	15.34	0.00	0.00	2.56	7.67				
10	9	10	-50.03	0.94	0.00	0.00	0.57	0.47				
11	10	11	-325.93	45.35	0.00	0.00	3.70	15.12				
12	10	12	96.02	0.30	0.00	0.00	0.39	0.13				
13	12	13	-3.98	0.00	0.00	0.00	0.02	0.00				
14	13	14	-193.56	0.81	0.00	0.00	0.79	0.48				
15	14	15	-616.28	7.36	0.00	0.00	2.52	4.09				
16	0	15	1,232.76	1.65	0.00	278.72	1.97	1.50				
17	15	16	616.48	3.03	0.00	0.00	1.75	1.68				
18	16	17	58.04	0.05	0.00	0.00	0.16	0.02				
19	17	18	-97.13	0.24	0.00	0.00	0.28	0.05				
20	18	19	-197.13	1.14	0.00	0.00	0.81	0.50				
21	19	3	-410.48	2.70	0.00	0.00	1.68	1.93				
22	4	20	31.88	0.45	0.00	0.00	0.36	0.20				
23	19	20	213.35	1.72	0.00	0.00	0.87	0.57				
24	20	28	115.74	7.56	0.00	0.00	1.31	2.22				
25	20	32	29.48	0.04	0.00	0.00	0.12	0.01				
26	32	17	-155.17	0.38	0.00	0.00	0.63	0.32				
27	32	33	84.65	3.11	0.00	0.00	0.96	1.24				
28	33	14	43.06	0.78	0.00	0.00	0.49	0.36				
29	33	16	-458.44	3.54	0.00	0.00	1.87	2.36				
30	28	33	-400.03	4.41	0.00	0.00	1.63	1.84				
31	30	14	-365.78	4.04	0.00	0.00	1.49	1.56				
32	31	13	-89.59	1.94	0.00	0.00	1.02	1.38				
33	10	31	79.87	2.24	0.00	0.00	0.91	1.12				
34	30	31	-69.46	1.29	0.00	0.00	0.79	0.86				
35	21	28	-69.04	1.28	0.00	0.00	0.78	0.85				
36	25	28	-161.53	3.09	0.00	0.00	1.83	4.12				
37	28	29	135.20	2.22	0.00	0.00	1.53	2.96				
38	26	29	-105.90	1.41	0.00	0.00	1.20	1.88				
39	29	30	-120.70	1.80	0.00	0.00	1.37	2.40				
40	27	30	-164.54	3.20	0.00	0.00	1.87	4.26				
41	26	27	-9.85	0.02	0.00	0.00	0.11	0.02				
42	25	26	63.36	0.55	0.00	0.00	0.72	0.73				
43	22	25	51.82	0.38	0.00	0.00	0.59	0.50				
44	23	26	-29.10	0.13	0.00	0.00	0.33	0.17				
45	24	27	-4.69	0.00	0.00	0.00	0.05	0.01				
46	24	23	30.64	0.14	0.00	0.00	0.35	0.19				
47	22	23	90.26	1.05	0.00	0.00	1.02	1.40				
48	21	22	132.81	1.43	0.00	0.00	1.51	2.87				
49	5	21	213.76	4.85	0.00	0.00	2.43	6.92				
50	7	22	159.27	6.42	0.00	0.00	1.81	4.01				
51	9	24	175.95	5.79	0.00	0.00	2.00	4.83				
52	0	8	602.06	23.55	0.00	302.15	6.83	47.10				
53	0	11	425.93	3.54	0.00	308.09	4.83	17.70				
Junction number												
1									969.0	150.0	47.3	1,078.2
2									970.0	0.0	47.0	1,078.4
3									972.0	100.0	43.8	1,073.1
4									974.0	100.0	41.2	1,069.1
5									985.0	100.0	34.5	1,064.7
6									977.0	100.0	38.0	1,064.7
7									980.0	100.0	36.7	1,064.8
8									982.0	0.0	41.9	1,078.6
9									975.0	100.0	38.2	1,063.3
10									948.0	100.0	50.4	1,064.2
11									964.0	100.0	63.1	1,109.6
12									932.0	100.0	57.2	1,063.9
13									923.0	100.0	61.1	1,063.9
14									919.0	100.0	63.1	1,064.7
15									903.0	0.0	73.3	1,072.1
16									920.0	100.0	64.6	1,069.0
17									948.0	0.0	52.4	1,069.0
18									962.0	100.0	46.5	1,069.2
19									972.0	0.0	42.6	1,070.4
20									972.0	100.0	41.9	1,068.6
21									978.0	150.0	35.5	1,059.8
22									975.0	150.0	36.1	1,058.4
23									968.0	150.0	38.7	1,057.3
24									965.0	150.0	40.1	1,057.5
25									969.0	150.0	38.6	1,058.0
26									965.0	150.0	40.1	1,057.5
27									962.0	150.0	41.4	1,057.5
28									966.0	150.0	41.2	1,061.1
29									962.0	150.0	42.0	1,058.9
30									954.0	150.0	46.2	1,060.7
31									937.0	100.0	54.2	1,062.0
32									958.0	100.0	47.9	1,068.6
33									937.0	100.0	55.7	1,065.5

*Metric conversions—15,839 gpm = 1 m^3/s, 3.28 ft = 1 m, 3.28 fps = 1 m/s, 0.145 psi = 1 kPa

The first approach involves the use of perturbation analysis, whereas the second approach involves an iterative cyclic procedure for the solution of Eq 2.

Perturbation analysis. Intuitively, one might suppose that the source contribution distribution for a particular junction node can be obtained using a perturbation analysis. In using perturbation analysis, a network computer model is first calibrated to model the existing distribution system for the demand patterns of interest. Once the model has been calibrated, the demand at an individual junction node is perturbed (increased or decreased) and the change in flow from all sources is computed. The contribution from each source is then assumed to be equal to the ratio of the incremental flow change (at the source) to the increase (or decrease) in flow at the junction node. The contribution factor for a particular source i and junction node j can thus be expressed as:

$$C(i,j) = (\Delta Q_i)/(\Delta D_j) \tag{8}$$

in which ΔQ_i is the net change in flow at source i, and ΔD_j is the change in demand at junction j.

The example network shown in Figure 1 can also be used to illustrate the use of perturbation analysis. For this example problem, the source flow distribution for junction 4 is desired. To predict this distribution, the demand at junction node 4 was increased to 13.9 L/s, and a new set of results was obtained. The incremental flows from each source that resulted from this increase in demand are shown in Table 4 along with the resulting contribution factors $C(i,4)$.

Although perturbation analysis can be used to predict the contribution factors in a straightforward manner, because of the complex operation of water distribution systems the resulting factors may not be correct. This can be demonstrated by considering the complete flow distribution based on the three sources (A, B, and C) as shown in Figure 2. This distribution was constructed from the results of the original network analysis and, for this simple example, the exact contribution factors are easily determined from the flow distribution. A comparison of the true contribution factors $[C*(i,j)]$ with the contribution factors obtained using perturbation analysis is shown in Table 4. As can be seen from the table, the results obtained using perturbation analysis contain significant error. As a result, it may be concluded that the use of a straightforward perturbation analysis may fail to yield the correct contribution factors.

Iterative cyclic procedure. As an alternative to perturbation analysis, an iterative cyclic solution to Eq 2 was also investigated. The form of Eq 2 suggests that the resulting matrix is strongly diagonally dominant and that such a procedure may yield a straightforward solution. To implement this procedure, Eq 1 is recast:

$$C(i,j) = \frac{Q_s(i) + \sum_{n \epsilon (j)}^{N_j} C(i,n)\, Q(n)}{Q_o(j)} \tag{9}$$

The contribution factors are solved starting at the first node and setting all undetermined concentrations to zero. As each node is solved, the updated values of previously determined contribution factors are utilized. After all concentrations are determined, the procedure is repeated until the values do not change.

For distribution networks that contain a sequence of dependent junction nodes (i.e., a source-determinant network), the contribution factors can be obtained in one pass if the junction nodes are evaluated in the proper order.[5] For example, determination of the contribution factors from source A by application of Eq 9 to the sequence of junction nodes 1, 2, 3, and 4 results in the following one-pass solution: $C(A,1) = 12.28/12.28 = 1$; $C(A,2) = 0.42C*(A,1)/8.04 = 0.052$; $C(A,3) = 5.44C*(A,2)/10.78 = 0.026$; and $C(A,4) = 5.55*C(A,1) + 2.6C*(A,2) + 4.47C(A,3)/12.62 = 0.460$.

Selection of some alternative solution sequence, such as solving junction nodes 4, 3, 2, and 1, would not have resulted in a one-pass solution but the same solution would have been obtained in successive applications. Indeed, it can be demonstrated that a feasible solution can be obtained regardless of the nodal solution sequence. The only potential difference between the solutions will be the number of iterations required.

Based on the previous observations, it may be argued that a more efficient algorithm could be developed by first locating the sequence of dependent nodes within the network so that Eq 9 could then be applied in one pass. It is anticipated that the algorithm required to locate an acceptable sequence could be

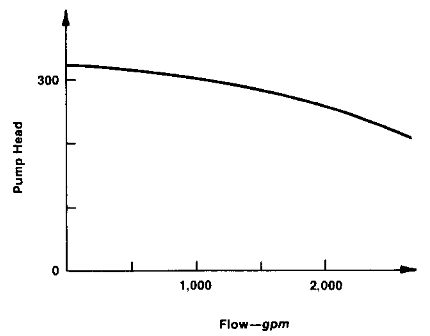

Figure 5. Well pump characteristics (*example 3*)

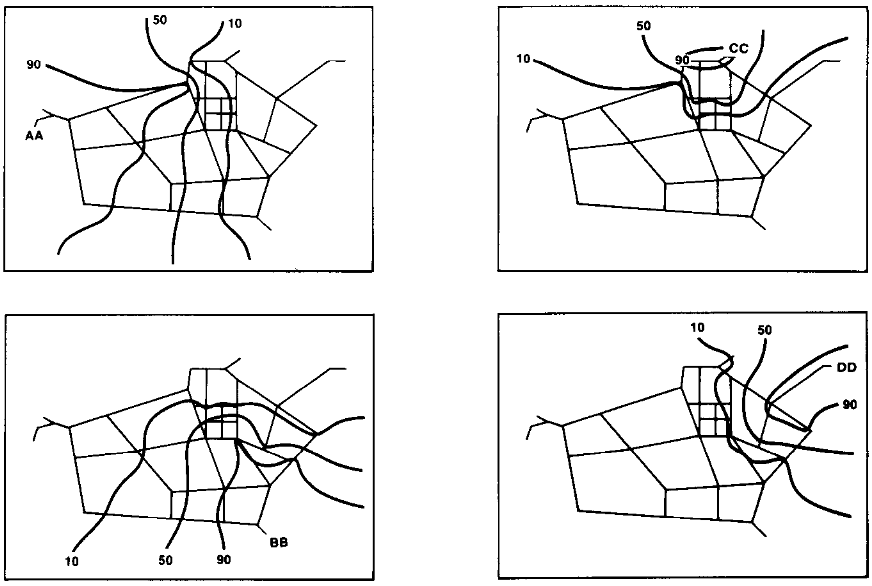

Figure 6. Example contribution contour plots

TABLE 7

Contribution factors for example network 3—percent

Node Number	Source			
	AA	BB	CC	DD
1	100.00	0.00	0.00	0.00
2	100.00	0.00	0.00	0.00
3	100.00	0.00	0.00	0.00
4	100.00	0.00	0.00	0.00
5	94.62	0.00	5.38	0.00
6	0.00	0.00	100.00	0.00
7	0.00	0.00	100.00	0.00
8	0.00	0.00	100.00	0.00
9	0.00	0.00	81.87	18.13
10	0.00	0.00	0.00	100.00
11	0.00	0.00	0.00	100.00
12	0.03	3.95	0.00	96.02
13	0.70	99.30	0.00	0.00
14	0.70	99.30	0.00	0.00
15	0.00	100.00	0.00	0.00
16	0.00	100.00	0.00	0.00
17	62.60	37.40	0.00	0.00
18	100.00	0.00	0.00	0.00
19	100.00	0.00	0.00	0.00
20	100.00	0.00	0.00	0.00
21	79.03	16.91	4.06	0.00
22	35.93	7.69	56.38	0.00
23	25.69	18.99	51.74	3.61
24	0.02	2.39	79.74	17.85
25	31.99	54.31	13.69	0.00
27	21.12	71.53	4.84	2.51
27	0.60	91.83	0.00	7.52
28	30.73	69.27	0.00	0.00
29	16.52	79.91	0.00	3.55
30	0.65	91.83	0.00	7.52
31	0.37	52.50	0.00	47.13
32	68.57	31.43	0.00	0.00
33	10.69	89.31	0.00	0.00

more computationally expensive than the few extra iterations required with the iterative approach. In addition, there is no guarantee that the distribution system will even contain such a sequence. For example, consider the simple distribution system shown in Figure 3. This distribution network is not source-determinant. As a result, it is impossible to find a sequence of junction nodes that will permit a one-pass solution of Eq 9. In this case, an iterative solution is the only way in which the contribution factors may be determined.

Application

A more realistic application of the iterative solution methodology may be obtained by consideration of the network shown in Figure 4. This system has 33 junction nodes, 53 pipes, 4 pumps, and 4 fixed-grade (supply) nodes and loosely represents an actual water distribution system. Supply nodes A and B have hydraulic grades of 800 ft (243.9 m), whereas supply node C has a fixed grade of 805 ft (245.4 m) and node D has a fixed grade of 795 ft (242.4 m). The supply nodes represent wells, and all four well pumps have the same characteristic curve (Figure 5). The remaining physical characteristics of the system are shown in Table 5. Because the data were already available in English units, these values are given, with the corresponding conversion of SI units noted.

The hydraulic results for this system are summarized in Table 6. Using these results and the cyclic procedure introduced in the previous section, a unique set of contribution factors was obtained for each junction node. The contribution factors for the entire system are presented in Table 7.

Graphical display of the results of contribution factor calculations are particularly useful for showing the influence of selected supplies. Contour plots of percentage values for contribution factors for each of the supplies for example 2 are shown in Figure 6. These plots clearly indicate the effects of each of the supplies on the network demands.

Conclusion

It is possible to explicitly determine the percentage of flow originating at various supply points at a specific location in a water distribution system. This procedure does require the solution of a set of simultaneous equations equal to the number of nodes in the distribution system. The equations utilize the results of a hydraulic analysis for the situation under investigation. As an alternative to a complete-matrix-solution technique, the equations may be solved one at a time using a cyclic-solution procedure. The resulting equations may be solved manually, with a short computer program, or may be set up and solved using a mathematical spreadsheet. The proposed iterative cyclic procedure has been applied to a number of networks, including ones up to 500 nodes and has been found to be a fast and efficient procedure for determining contribution factors for all the networks tested.

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