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SUMMARY

A digital distributed parameter model was used to study the effects of propellant-feed-system coupling and various hydraulic parameters on the analytical prediction of chugging instabilities. Coupling between the combustion chamber and feed system was controlled by varying the compliance of the injector-dome region. The coupling with the feed system above the pump was varied by changing the amount of cavitation compliance at the pump inlet. The stability limits and chugging frequencies proved to be strongly dependent on the degree of feed-system coupling. The maximum stability condition occurred with intermediate coupling.

Under conditions of a high degree of feed-system - combustor coupling, the stability limits and chugging frequencies were primarily dependent on the feed-system characteristics; the responses were characterized by beating patterns. For the system analyzed, the pump suction line had little effect on the stability limits or chugging frequencies. Beating, present under the condition of near zero injector-dome compliance, was eliminated when the suction line was decoupled by employing a sufficiently high value of pump-inlet compliance.

Under conditions of maximum feed-system coupling, the magnitude and distribution of line losses in the discharge line had a significant effect on the stability limits but had negligible effect on the chugging frequency and beating characteristics. Also, the length of the discharge line greatly affected the stability limits, chugging frequency, and beating characteristics. The length of the suction line, however, had little effect on the stability limits and chugging frequency but did influence the beating pattern.

A resistive-shunt device attached to the pump discharge line to suppress chugging was investigated. The analysis showed that the device was effective under conditions of high feed-system coupling.

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INTRODUCTION

Analyses of chugging-type instabilities that occur in liquid-rocket engines sometimes ignore completely the dynamic characteristics of the propellant feed system (e.g., ref. 1). When the feed-system dynamics are considered in a stability analysis, it is common to "lump" parameters and to linearize the impedance characteristics of the system (refs. 2 to 6). Distributed parameter treatments of the feed system in chugging studies are rare. A linearized distributed parameter analysis of a simple pressure-fed system is given in reference 7.

Simplified feed-system models have been employed because the inclusion of a realistic representation of the propellant feed system considerably complicates a chugging-stability analysis. For example, it is difficult to obtain an accurate solution of the partial differential equations with an analog computer because of the large number of network elements required to approximate a distributed parameter feed system at chugging frequencies (100 to 500 cps).

The importance of the type of a feed-system model chosen for a chugging-stability analysis depends on the degree of coupling that exists between the feed system and the combustion chamber. Compliance (structural and fluid) in the dome region immediately preceding the injector provides a natural mechanism for varying the amount of coupling between the feed system and the combustor. When the dome compliance is high, the feed system is decoupled and can be approximated by assuming constant pressure upstream of the injector orifices, as was done in reference 1. Conversely, for a hard dome (very low compliance), a high degree of coupling can occur, and the feed-system parameters would have a large influence on the system stability, as shown in reference 7. Since the dome compliance usually falls between these two extremes, a realistic distributed parameter model of the feed system should be employed.

In the analytical study discussed in this report, a nonlinear distributed parameter model of a hypothetical pump-fed propellant system was employed, along with a finite-difference equation for the combustion chamber, to evaluate the effect of the degree of combustor-feed-system coupling and various feed-system parameters on the prediction of stability limits, chugging frequencies, and the response characteristics of a rocket engine. The propellant-system model employed (refs. 8 and 9) was developed at the Lewis Research Center for the analysis of pressure and flow disturbances in liquid-rocket feed systems.

The hypothetical system analyzed (shown in fig. 1) represents a monopropellant (or one side of a bipropellant) feed system plus a combustion chamber. The effect of combustor - feed-system coupling was studied by systematically varying the compliance of the injector-dome region. Parametric studies were made for the hard-dome case, where the feed-system parameters have a maximum influence on the predicted stability

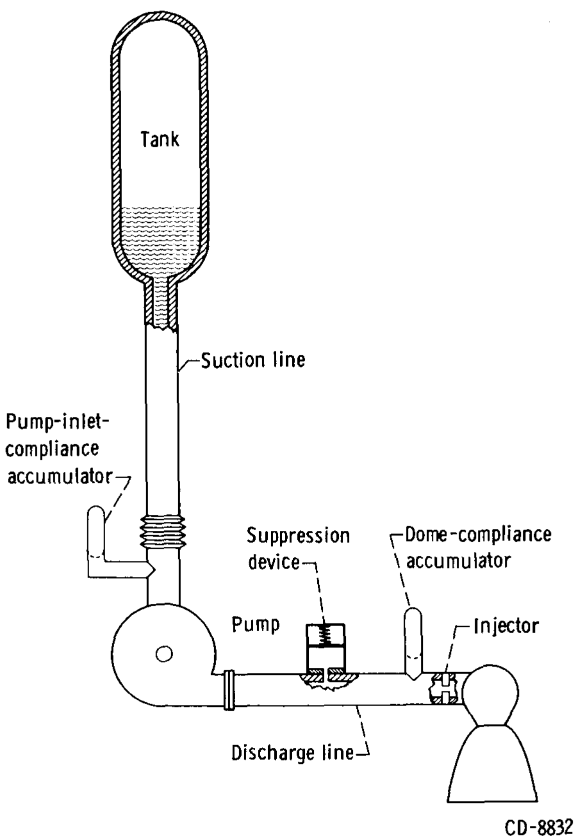


Figure 1. - Schematic drawing of feed system.

limits and response characteristics. Parameters included the effect of the magnitude and location of viscous resistance simulators (orifices) in the pump discharge line, the lengths of both the pump suction and discharge lines, and the degree of suction-line coupling. The suction-line coupling was controlled by varying the amount of cavitation-induced compliance of the pump inlet. Finally, a suppression device to improve the stability of the system was studied.

ANALYSIS

Feed-System-Combustor Model

The system studied (fig. 1) consists of a monopropellant (or one side of a bipropellant) feed system plus a rocket-engine combustion chamber. The feed system consists of a tank, suction line, pump, discharge line, and a nonlinear (square law) injector. Viscous losses in the suction and discharge lines are simulated by distributing a number of square-law orifices in each line. The pump-inlet cavitation-induced compliance and the injector dome compliance are simulated by accumulators, as shown in figure 1. The pump is assumed to be operating near its design point. The relation between the static pressure-head rise across the pump ΔH_p and the volume flow rate Q is expressed by the following quadratic equation

$$\Delta H_p = A + BQ + CQ^2 \tag{1}$$

where A, B, and C are coefficients.

The combustion chamber is represented by the conventional relation

$$\frac{d\left[H_c(t)\right]}{dt} + \frac{H_c(t)}{\theta_g} = \frac{C^*}{A_T g \theta_g} Q_i(t - \tau) \tag{2}$$

where H_c is the chamber pressure head (in feet of propellant), Q_i the injector flow rate, θ_g the gas residence time, A_T the throat area of the engine nozzle, C^* the characteristic velocity, t the time, and τ the combustion dead time. This equation is utilized in the analysis by expressing the time-derivative of the chamber pressure in finite-difference form.

The combustor and feed system (fig. 1) was analyzed by the wave-plan method of references 8 and 9. The digital computer program of reference 9 was employed with only minor modifications and additions. Because of the high frequencies (100 to 500 cps) encountered in this study, a second-order backward finite-difference form of the combustion-chamber-pressure relation (eq. (2)) was employed in the calculations. This relation is given in appendix A as equation (A2).

Chugging-Suppression Device

In addition to various parametric studies, an analysis was made of the effect of a chugging-suppression device on the stability of the combustor and feed system of figure 1 under highly coupled conditions. The device was attached to the midpoint of the discharge line and consisted of an accumulator that was assumed to be compliant and to have a linear resistance at the entrance. The shunt device affected the dynamic response and had no effect on the steady-state hydraulic characteristics of the feed system.

The mechanism of a chugging-suppression device is based on the following reasoning: Chugging can occur only under conditions for which the impedance (looking upstream) at the injector is very small. The characteristic impedance of the discharge line, however, is considerably larger than the combustion resistance $\partial H_c/\partial Q_i$. The feed system must therefore be in a lightly damped resonant mode in order for a low impedance to occur at the injector. By placing a resistive-shunt device, with an impedance somewhat less than characteristic, at a distance of an appreciable fraction of a wavelength from the injector, additional damping is provided. The increased damping prevents the occurrence of a very low minimum of feed-system impedance at the injector and, therefore, suppresses chugging.

The equations used to represent the suppression device are derived in wave-plan form in appendix B. These equations were programed and added to the routines of references 8 and 9.

System Parameters

The numerical values of the major feed-system and combustion-chamber parameters are as follows:

Line lengths, ft:	
Feed line	40
Discharge line	6
Line areas, ft^2:	
Feed line	0.2
Discharge line	0.05
Acoustic velocities, ft/sec:	
Tank	1000
Feed line	2000
Discharge line	3000
Propellant density, lb/ft^3	53
Mean flow rate, ft^3/sec	3
Pump coefficients:	
A	1300
B	650
C	-115
Combustion-chamber pressure head, ft	1977
Combustion parameters, msec:	
Combustion dead time	1
Gas residence time	1

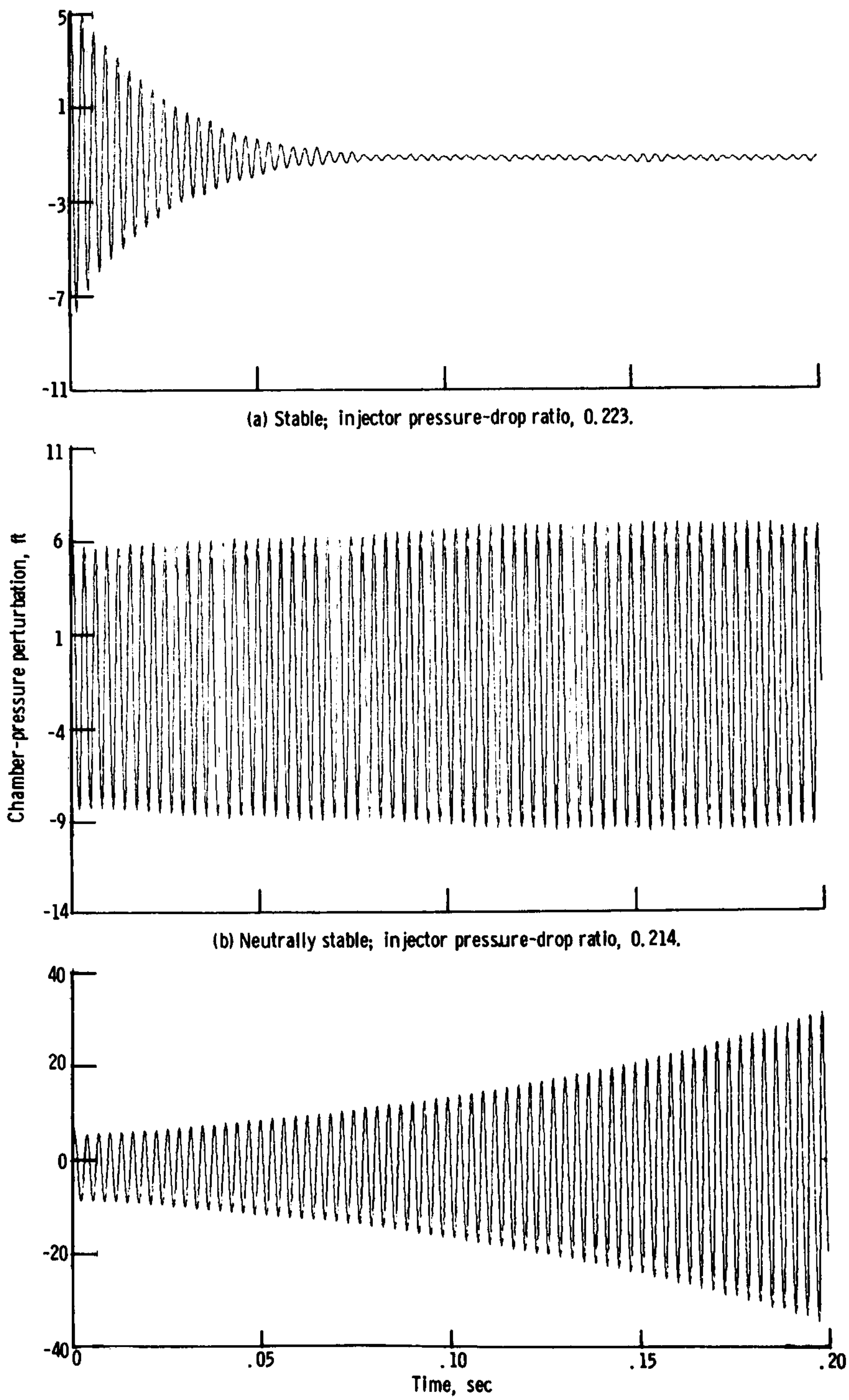
The values listed are for the reference system, and, unless otherwise specified for a particular parametric study, these values were used in the numerical calculations.

Method of Analysis

The combustion-chamber parameters (and dynamic characteristics) were held constant throughout the study. The quantity $C^*/A_T g\theta_g$ in equation (2) was equal to approximately 659 000 per square foot.

The feed-system variables employed in the parametric studies included injector pressure drop, dome compliance, pump-inlet compliance, suction-line length, discharge-line length, magnitude of discharge-line pressure loss, and the number and location of orifices used to represent analytically the discharge-line pressure loss.

The analysis for a given set of feed-system parameters was conducted as follows: The mean flow rate was defined at the reference value of 3 cubic feet per second. With this flow rate in the system, the tank ullage pressure was adjusted a small amount (as necessary) to keep the mean combustion-chamber pressure head at 1977 feet (728 psi) for all cases. With the system running (on the computer) at steady state, a small step change (0.4 percent) in chamber pressure was introduced, and the resulting pressure and flow



(a) Stable; injector pressure-drop ratio, 0.223.

(b) Neutrally stable; injector pressure-drop ratio, 0.214.

(c) Unstable; injector pressure-drop ratio, 0.209.

Figure 2. - Combustion-chamber responses for uncoupled system.

responses at various points in the feed system were computed. The stability characteristics of the combustor - feed system were determined by noting whether the amplitude of the disturbances increased, died out, or was maintained at a constant level (fig. 2). The combustion-chamber pressure was usually monitored, and this response, in many cases, was plotted directly by a digital plotter.

RESULTS AND DISCUSSION

Effect of Feed-System Coupling

The degree of coupling between the propellant feed system and combustion chamber (fig. 1) was varied by changing the compliance of the injector-dome accumulator. For each value of dome compliance employed, the response of the system to a 0.4-percent step change (imposed for 0.2 msec) in combustion-chamber pressure was determined for a range of injector pressure drops. The introduction of a step change in combustion-chamber pressure is analogous to the experimental technique of bombing the combustion chamber to initiate chugging oscillations.

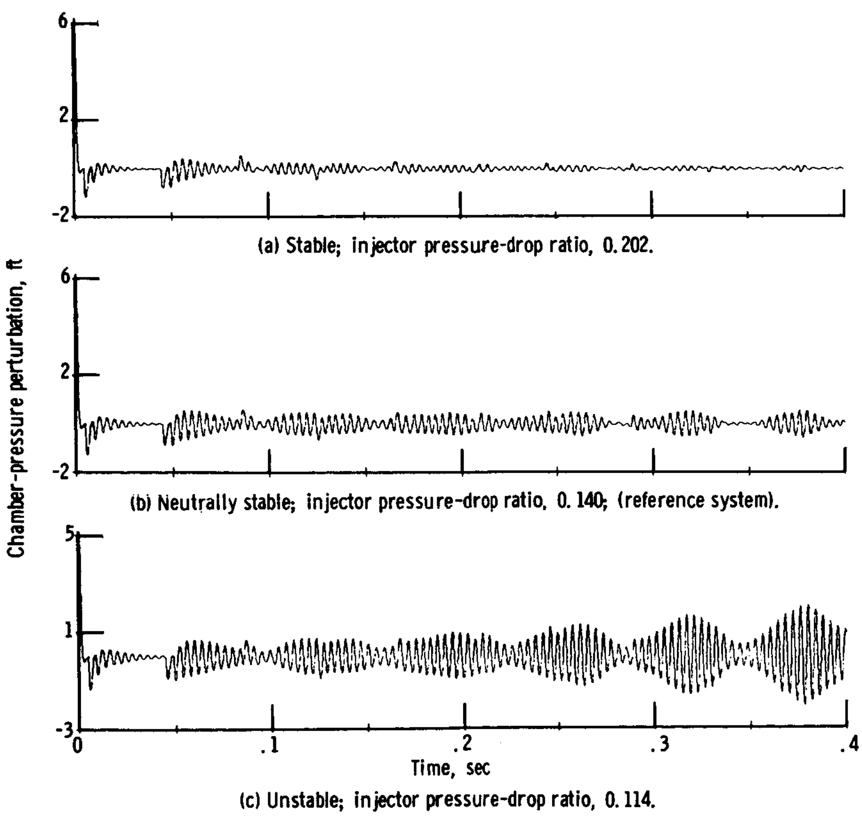


Figure 3. - Combustion-chamber responses for coupled system.

Examples of combustion-chamber responses to the step change, in the form of pressure-head traces from the digital plotter, are shown in figures 2 and 3. Figure 2 contains a stable, a neutrally stable, and an unstable response for the uncoupled case (very large dome compliance). The pressure-time traces in figure 2[1] show that, when there is negligible feed-system coupling, small changes in injector resistance have a large effect on the system response. This results from the fact that, as the feed-system coupling becomes small, the injector offers the principal resistance to the chugging oscillations. Figure 3 contains a stable, a neutrally stable, and an unstable response for the case of maximum feed-system coupling (zero dome compliance), where relatively large changes in injector resistance produce only small changes in the system response.

TABLE I. - EFFECT OF DOME COMPLIANCE ON NEUTRAL STABILITY CONDITION

Dome compliance		Injector pressure-drop ratio	Chugging frequency, cps
ft^3/ft	$in.^3/psi$		
2.5×10^{-3}	11.7	0.214	322
5×10^{-5}	.234	.207	327
5×10^{-6}	.0234	.160	370
2.5×10^{-6}	.0117	.094	416
1.5×10^{-6}	.0070	.055	455
5×10^{-7}	.00234	.114	263
0	0	.140	260

The neutral stability condition was determined for a large range of injector dome compliances. The ratio of injector pressure drop to chamber pressure (hereinafter called injector pressure-drop ratio) needed for neutral stability at various values of dome compliance is given in table I along with the corresponding chugging frequency.

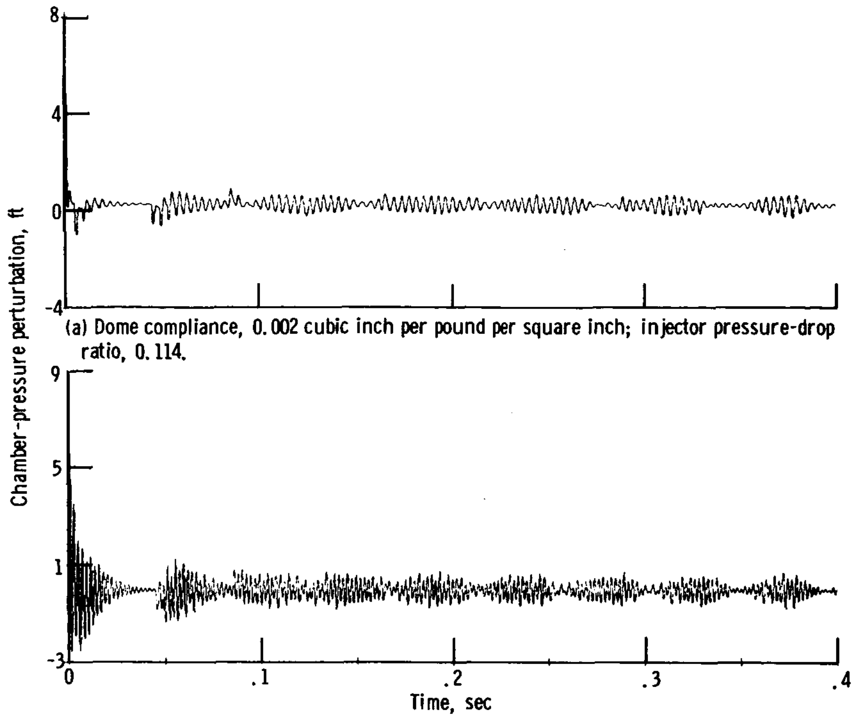
Examples of calculated combustion-chamber pressure responses at (or near) the neutral stability condition are contained in figures 2 to 4. Figure 2(b) shows a neutrally stable response for the highly uncoupled (maximum compliance case of table I) feed system. For a completely uncoupled feed system (constant pressure upstream of the injector), the chugging frequency is controlled by the combustion-chamber parameters and given closely by a solution of the implicit equation

$$\pi = \tan^{-1}(2\pi f\theta_g) + 2\pi f\tau$$

where f is the chugging frequency, τ is the combustion dead time, and θ_g is the gas residence time. The solution of this equation gives a frequency of 323 cps compared with 322 cps obtained from the response shown in figure 2(b) and listed in table I.

The two combustion-chamber pressure traces shown in figure 4 are for intermediate degrees of coupling. There is evidence of two frequencies in the response of figure 4(b). These frequencies are approximately 250 cps, which is caused by the excitation of the half-wave mode of the pump discharge line, and 450 cps, which is the chugging frequency associated with this value of dome compliance (0.007 $in.^3/psi$).

[1]The pressure-head scale used in figures 2 to 4 and 6 to 11 is the difference between the head at time t and the steady-state value (i.e., chamber-pressure perturbation).



(a) Dome compliance, 0.002 cubic inch per pound per square inch; injector pressure-drop ratio, 0.114.

(b) Dome compliance, 0.007 cubic inch per pound per square inch; injector pressure-drop ratio, 0.055.

Figure 4. - Neutrally stable combustion-chamber responses for partially coupled systems.

The pressure-time traces of figures 3 and 4 show that when there is a significant amount of feed-system coupling, beats occur in the oscillatory response. For the case of maximum coupling (zero dome compliance), the beats are quite strong (fig. 3). The beating appears to occur because more than one of the higher order acoustic modes of the suction line are near the chugging frequency (which for this system is near the 250-cps half-wave frequency of the discharge line). The excitation of these higher suction-line modes, which are close together in frequency (within 10 percent), results in beats.

The data of table I, shown graphically in figure 5, depict the stability limit for this system as a function of injector-dome compliance over the range studied. Figure 5 points out that there is an optimum dome compliance ($\approx$0.0075 in.^{3}/psi) from the standpoint of system stability. This optimum compliance value is influenced significantly by feed-system parameters. Also, the effect of feed-system coupling on the stability limit is strong below an injector dome compliance of 0.1 cubic inch per pound per square inch. It is likely that the natural compliance of the dome region for a system having these hydraulic characteristics would be lower than this value.

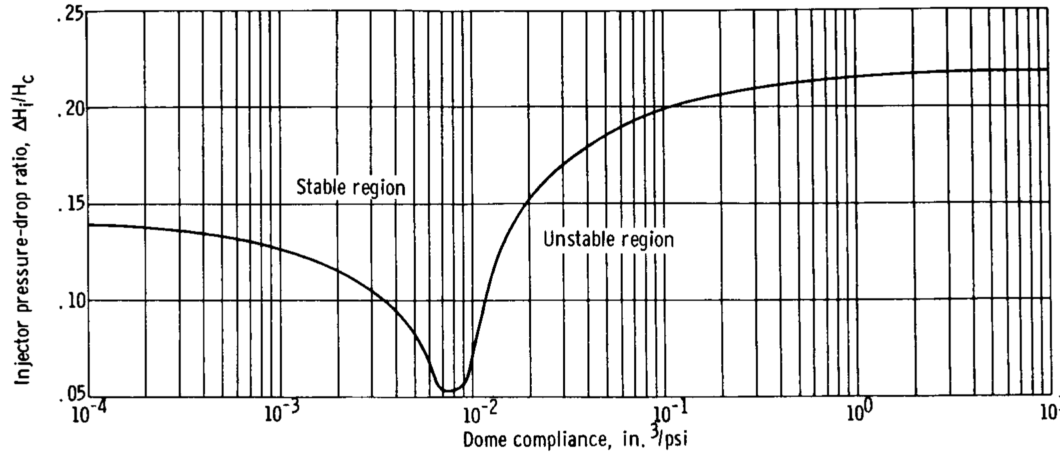


Figure 5. - Effect of dome compliance on injector pressure-drop ratio needed for neutral stability.

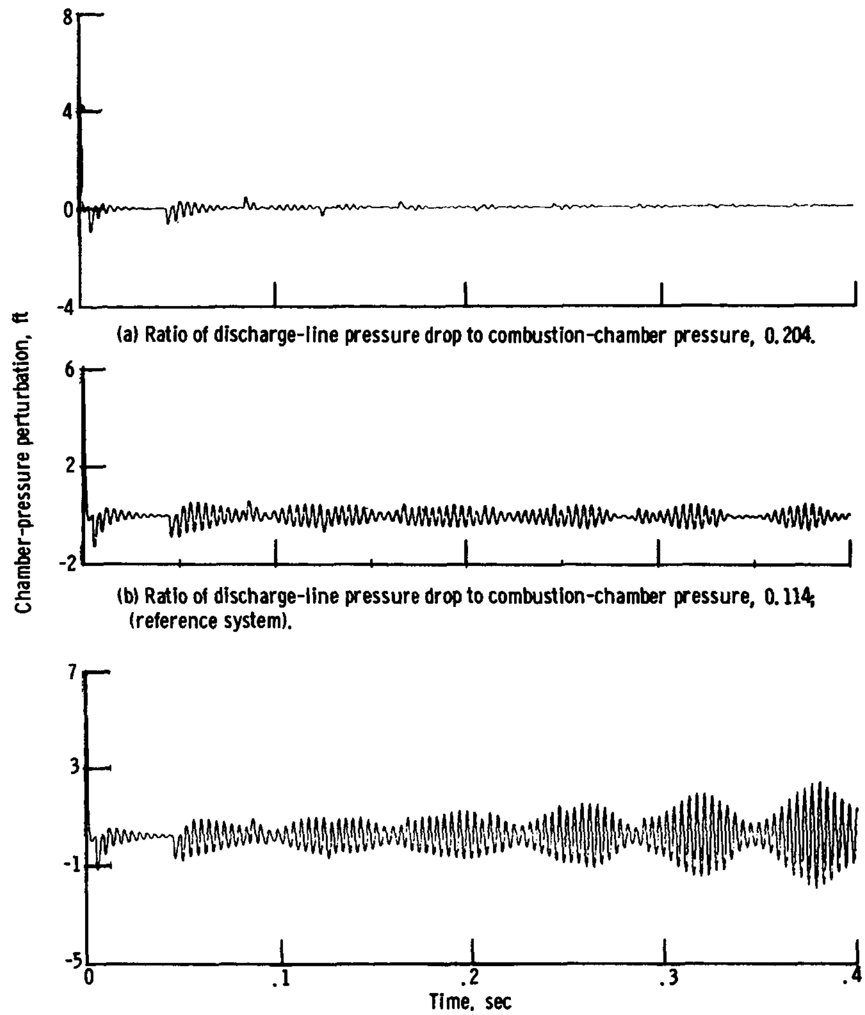
Effect of Feed-System Parameters

Under conditions of high degrees of feed-system coupling, feed-system parameters affect the calculated stability limits, chugging frequencies, and various response characteristics. Parametric studies were conducted in order to evaluate some of these effects.

Magnitude of discharge-line losses. - The magnitude of the discharge-line pressure loss affected the stability limit for the system significantly. In the analytical calculations, the line losses were normally lumped at nine points along the discharge line. Increases in the magnitude of the discharge-line loss increased the stability; chugging frequencies, however, were not significantly affected. This is illustrated in figure 6 which shows three cases computed at the same injector pressure-drop ratio but with different magnitudes of discharge-line pressure drop.

Distribution of discharge-line losses. - The location and number of points at which the discharge-line losses were lumped in the analysis also significantly affected the stability limits but not the chugging frequency. Figure 7 shows three cases for the same total discharge-line loss lumped at one, four, and nine points along the line. The injector pressure-drop ratio was the same for each case. A comparison of figures 6(c) and 7(a) shows that, for this system, lumping the discharge-line resistance at the midpoint of the discharge line in the analytical calculation is practically the same as neglecting the losses entirely. Thus, both the magnitude and location of discharge-line losses may significantly affect the stability limits.

Pump-inlet compliance. - The inlet of the pump often represents a region of locally high fluid compliance due to the presence of cavitation. The pump-inlet compliance (represented by pump-inlet accumulator in fig. 1, p. 3) provides a mechanism for decoupling the suction line from the discharge line. This can range from complete suction-



(c) Ratio of discharge-line pressure drop to combustion-chamber pressure, 0.031.

Figure 6. - Effect of magnitude of discharge-line pressure drop. Injector pressure-drop ratio, 0.140.

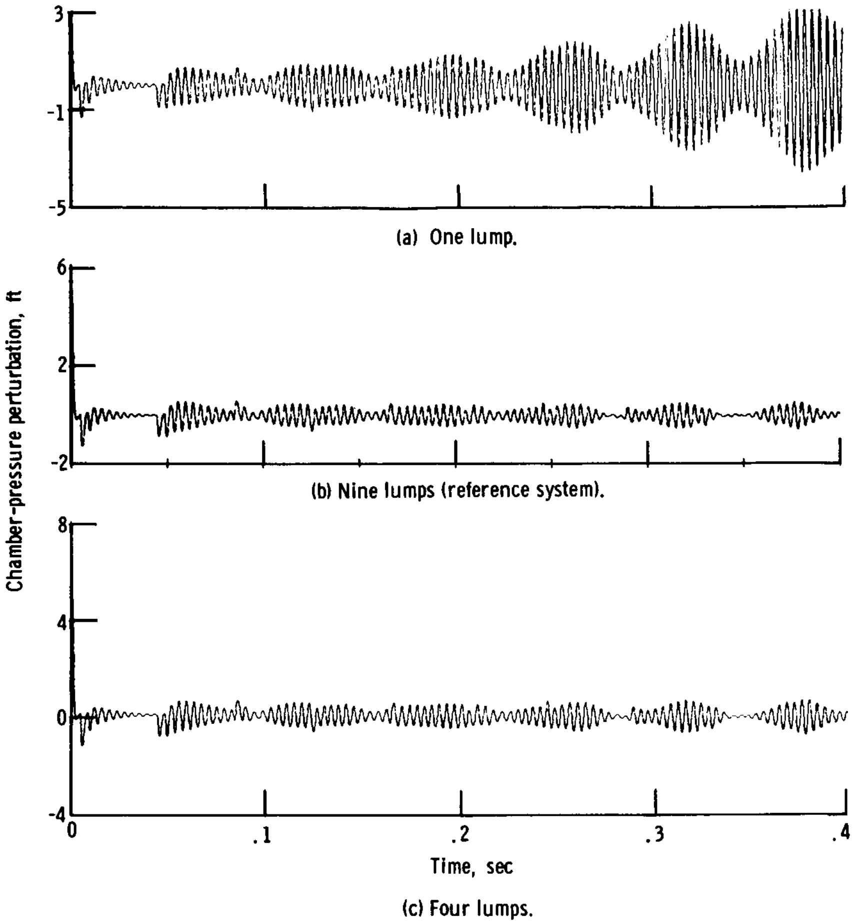
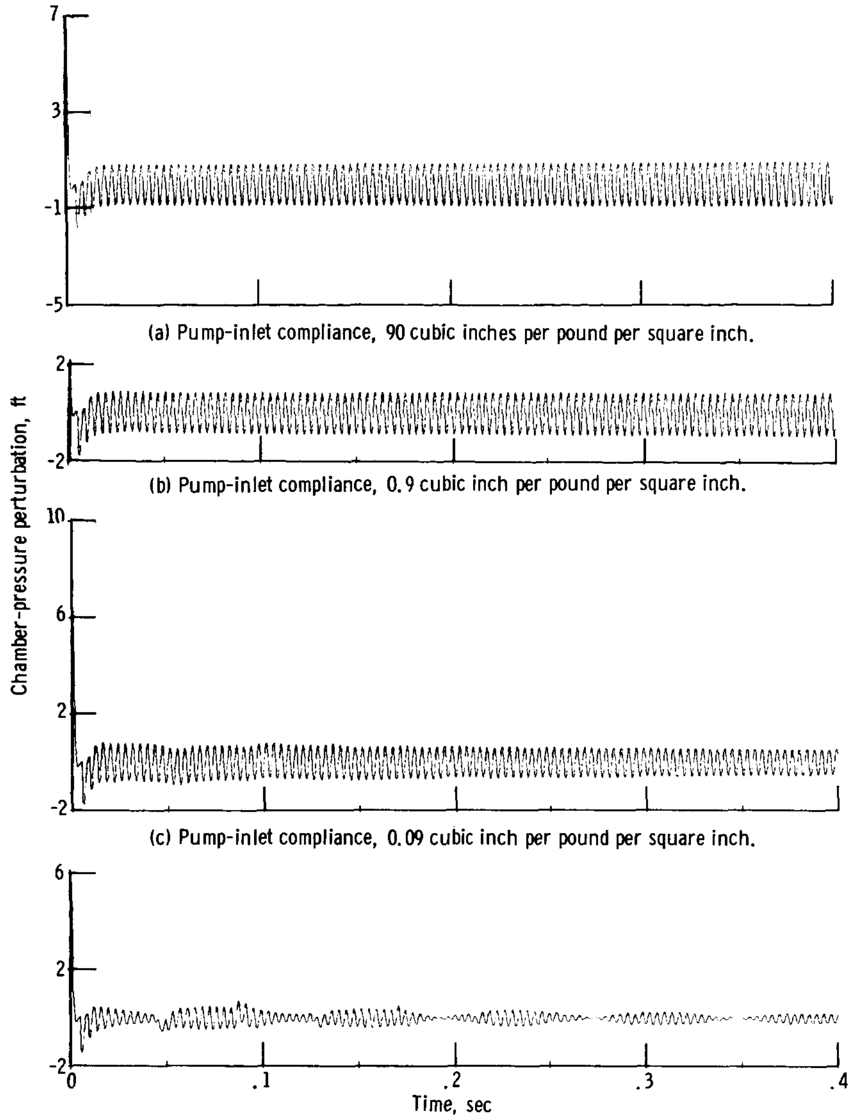


Figure 7. - Effect of distribution of discharge-line losses. Injector pressure-drop ratio, 0.140; ratio of discharge-line pressure drop to combustion-chamber pressure, 0.114.

line coupling (zero cavitation compliance) to effective uncoupling (large cavitation compliance).

For this system, the stability limit and chugging frequency were insensitive to the amount of suction-line coupling. Figure 8 shows four cases computed with the same value for the injector pressure-drop ratio (which is close to the value needed for neutral stability for each case) and for different values of pump-inlet compliance. These chamber-pressure responses may be compared with figure 6(b) or 7(b) which are for the same case with no cavitation compliance at the pump inlet. The most significant result of these calculations is the dependence of the beating characteristics on pump-inlet compliance. This dependence ranged from the absence of beats at high pump-inlet compliance to strong beats at zero inlet compliance. Thus, the premise is strengthened that the beating is caused by exciting two or more of the higher modes of the suction line that are close to the response frequency of the system.

Suction-line length. - The length of the pump suction line was varied, and the response of the chamber pressure was determined for the condition of maximum feed-system coupling. The neutral stability point and the chugging frequency were only



(a) Pump-inlet compliance, 90 cubic inches per pound per square inch.

(b) Pump-inlet compliance, 0.9 cubic inch per pound per square inch.

(c) Pump-inlet compliance, 0.09 cubic inch per pound per square inch.

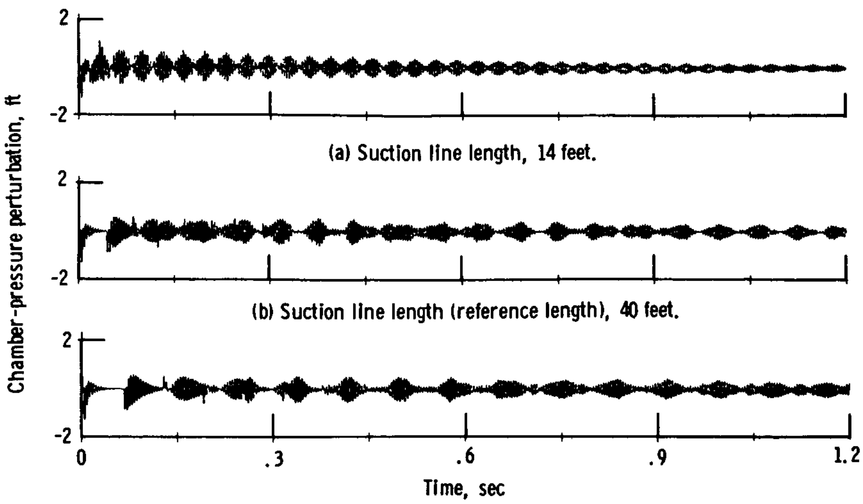
(d) Pump-inlet compliance, 0.009 cubic inch per pound per square inch.

Figure 8. - Effect of pump-inlet compliance. Injector pressure-drop ratio, 0.140.

slightly dependent on suction-line length. The beating characteristics were affected, however, as shown for three cases in figure 9. The system responses are given for a relatively long period of time (1.2 sec). Figure 9 again demonstrates the influence of the suction line on the occurrence of beating.

Discharge-line length. - The length of the discharge line was varied while both the total line loss and the injector pressure-drop ratio were held constant. This affected the stability limits, the chugging frequency, and the beating characteristics of the system. The results for an injector pressure-drop ratio of 0.073 are summarized in table II.

Examples of combustion-chamber-pressure responses for several discharge-line lengths are shown in figure 10. Table II and figure 10 indicate that the system stability



(c) Suction line length, 62 feet.

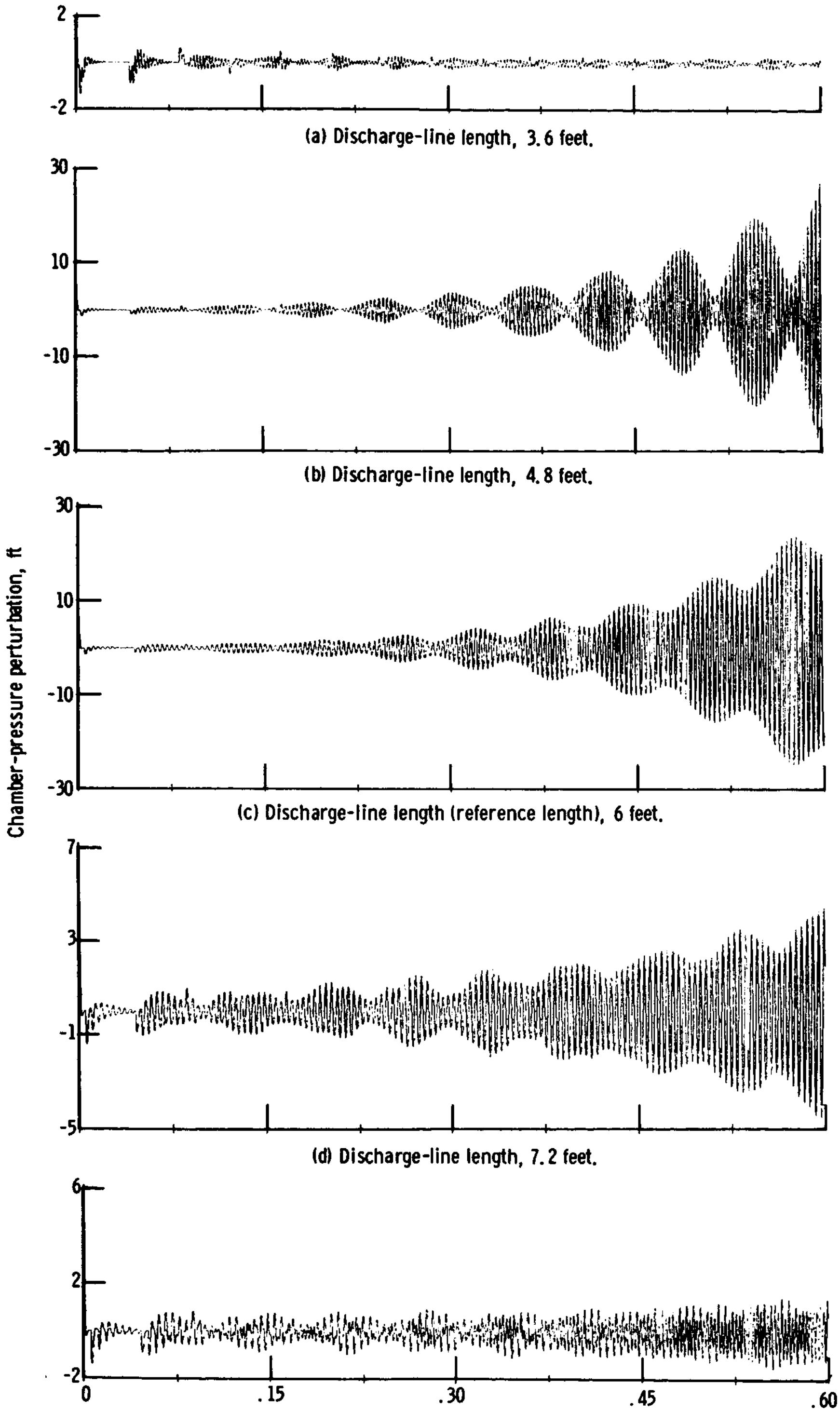
Figure 9. - Effect of suction line length. Injector pressure-drop ratio, 0.140.

TABLE II. - EFFECT OF PUMP-DISCHARGE-LINE LENGTH ON SYSTEM RESPONSE

[Injector pressure-drop ratio, 0.073.]

Discharge-line length, ft	Chugging frequency, cps	Beat period, msec	Maximum peak-to-peak pressure-head perturbation amplitude after 0.6 sec, ft
3	416	(a)	0.33
3.6	385	50	.8
4.2	345	53	31
4.8	294	56	66
5.4	276	58	75
6	255	59	30
6.6	233	63	16.3
7.2	220	70	6.7
7.8	203	60	1.9
8.4	357	(a)	2.0
9.0	328	75	3.5

[a]No definite beating pattern.



(a) Discharge-line length, 3.6 feet.

(b) Discharge-line length, 4.8 feet.

(c) Discharge-line length (reference length), 6 feet.

(d) Discharge-line length, 7.2 feet.

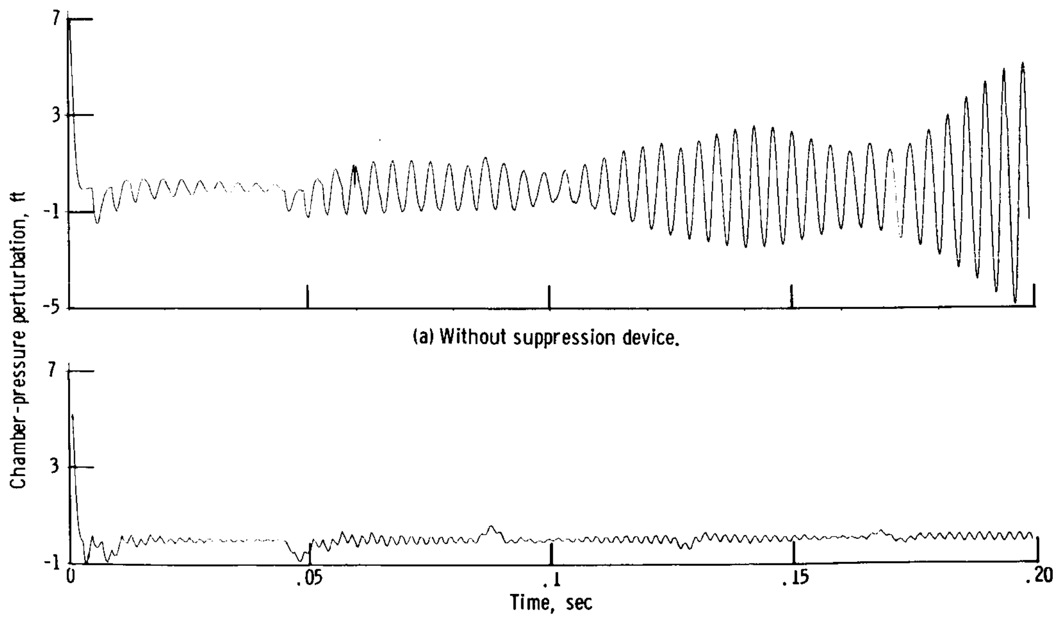
(e) Discharge-line length, 8.4 feet.

Figure 10. - Effect of discharge-line length. Injector pressure-drop ratio, 0.073.

is considerably influenced by the length of the pump discharge line. This system appears to have maximum instability with a discharge-line length of 5.4 feet, and it appears to be completely stable with a line length of 3 feet.

Effect of Chugging-Suppression Device

The stability of the reference combustor - feed system (with zero dome compliance) was determined with the chugging-suppression device attached to the midpoint of the discharge line. The resistive-shunt device had a resistance of 620 feet per cubic foot per second and a compliance of 0.8×10^{-6} cubic foot per foot. The neutral stability condition for the reference system without the suppression device occurred at an injector pressure-drop ratio of 0.140. With the suppression device attached to the pump discharge line, the system was effectively stabilized to much lower values of injector pressure-drop ratio. As an example, figure 11 presents digital plotter traces for the combustion-chamber pressure computed with and without the suppression device for an injector pressure-drop ratio of 0.05. Even at this low injector pressure-drop ratio, the system, with the suppression device attached, is near the neutral stability condition. Examination of the pressure trace in figure 11(b) shows that the 250-cps chugging instability evident in the trace of figure 11(a) is effectively suppressed by the device. The small remaining disturbance in figure 11(b) corresponds to the next higher discharge-line mode, namely, the full-wave, 500-cps resonance.



(b) Suppression device. Resistance, 620 feet per cubic foot per second; compliance, 0.8×10^{-6} cubic foot per foot.

Figure 11. - Effect of suppression device attached at midpoint of discharge line. Injector pressure-drop ratio, 0.05.

The small instability appearing at 500 cps (in spite of the suppression device) can be explained as follows: As a pressure-sensitive device, the accumulator is ineffective if located at a point in the line where the pressure perturbation approaches zero. This condition may exist at a frequency that makes the distance from the injector to the suppression device a half wavelength (or multiple). If this frequency coincides with one of the lightly damped, natural frequencies of the feed system and if it falls in the range in which the combustion-chamber impedance is negative, an instability can occur. This circumstance was overlooked when the device was located precisely at the midpoint of the discharge line (in order to suppress the 250-cps mode) and accounts for the presence of the 500-cps instability in the trace of figure 11(b). This difficulty can be overcome by locating the accumulator at a position slightly less than a half wavelength from the injector at the highest frequency of the first interval in which the combustion chamber impedance is negative.

CONCLUDING REMARKS

It has been shown by an example that consideration of propellant-feed-system dynamics can significantly affect the analytical prediction of stability limits, chugging frequencies, and other response characteristics for liquid-rocket engine systems. The importance of the feed-system dynamics in any given stability analysis is dependent on the amount of compliance (fluid and structural) present in the injector-dome region of the system being modeled. When the dome compliance is sufficiently high, the feed system will be effectively uncoupled from the combustion chamber, and consideration of feed-system dynamics in a chugging stability analysis is not necessary. In many practical situations, however, it is likely that the injector dome will be sufficiently hard that feed-system dynamics will be an important consideration. In these cases, omission of a detailed feed-system model from the stability analysis may lead to an over-conservative design.

The example of this report demonstrates that the wave-plan method provides a useful analytical tool for a distributed parameter representation of a propellant feed system at chugging frequencies. It also indicates that certain simplifications of feed-system geometry may be possible when modeling a particular system - regardless of the analytical technique used. In the system analyzed in this report, the feed system above the pump could have been neglected with no significant change in the predicted stability limits or chugging frequencies. It is likely that this would be true for many operational systems. Also, the presence of pump-inlet cavitation in operational systems would most likely preclude the excitation of the higher suction line resonant modes (which result in beats). In pressure-fed propellant systems (no turbopump), the higher modes may appear in the system response if the feedline is sufficiently long.

The results of this study further indicate that, for systems with sufficiently hard domes, the designer may be able to improve the stability substantially by small variations in certain discharge-line hydraulic parameters. In addition, resistive-shunt or other types of chugging-suppression devices can be incorporated into a system and their effect readily analyzed by the methods presented in this report.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 14, 1966,
128-31-06-09-22.

APPENDIX A

FINITE-DIFFERENCE FORM OF COMBUSTION-CHAMBER EQUATION

The equation (eq. (17) of ref. 9) relating combustion chamber pressure to flow through the injector is

$$\frac{d\left[H_c(t)\right]}{dt} + \frac{H_c(t)}{\theta_g} = \frac{C^*}{A_T g \theta_g} Q_i(t - \tau) \tag{2}$$

where H_c is the chamber pressure head, Q_i the injector flow rate, θ_g the gas residence time, A_T the throat area of the engine nozzle, C^* the characteristic velocity, and τ the dead time. In order to utilize this equation in the wave-plan analysis, it is necessary to express the derivative of the chamber pressure in finite-difference notation in terms of the working time interval Δt. A second-order backward finite-difference representation for the derivative is

$$\frac{d\left[H_c(t)\right]}{dt} = \frac{3H_c(t) - 4H_c(t - \Delta t) + H_c(t - 2\ \Delta t)}{2\ \Delta t} \tag{A1}$$

Substituting this relation into equation (2) gives

$$H_c(t) = \frac{\dfrac{C^*}{A_T g \theta_g} Q_i(t - n\ \Delta t) + \dfrac{4H_c(t - \Delta t) - H_c(t - 2\ \Delta t)}{2\ \Delta t}}{\dfrac{1}{\theta_g} + \dfrac{3}{2\ \Delta t}} \tag{A2}$$

where n represents the number of working time intervals Δt making up the engine dead time. This equation is used to calculate chamber pressure in the injector subroutine of the digital program.

APPENDIX B

WAVE-PLAN REPRESENTATION OF SUPPRESSION DEVICE

The chugging-suppression device is a shunt accumulator (fig. 1, p. 3) assumed to be compliant and to have a linear resistance at the entrance. This device is attached to the discharge line. Figure 12 shows conditions that exist before and after wave action at an accumulator attached at an interior point of a liquid transmission line. The accumulator volume flow rate before wave action is Q_1. After wave action, it is given by

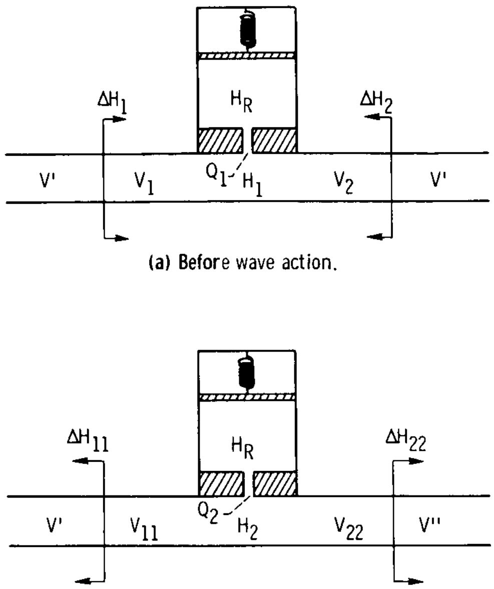
$$Q_2 = K(H_2 - H_R) \tag{B1}$$

where Q_2 is the new volumetric flow into the accumulator, K is the reciprocal of the linear accumulator resistance, and H_2 is the line pressure head adjacent to the accumulator. The accumulator pressure head H_R changes little during wave reflection.

The equation of continuity after wave action is

$$Q_2 = A_D(V_{11} - V_{22}) \tag{B2}$$

where A_D is the area of the discharge line and V_{11} and V_{22} are the velocities in the line to the left and right of the accumulator, respectively.



(a) Before wave action.

(b) After wave action.

Figure 12. - Conditions at suppression device.

The momentum equations across the incoming and outgoing waves may be combined to give

$$\Delta H_{11} = \Delta H_1 + \frac{C}{g}(V_1 - V_{11}) \tag{B3}$$

$$\Delta H_{22} = \Delta H_2 + \frac{C}{g}(V_{22} - V_2) \tag{B4}$$

where C is the discharge-line wave velocity; ΔH_1, ΔH_{11}, ΔH_2, and ΔH_{22} are the pressure-wave magnitudes; and V_1 and V_2 are the velocities in the line before wave action. The line pressure after wave action is given by

$$H_2 = H_1 + \Delta H_1 + \Delta H_{11}$$

$$= H_1 + \Delta H_2 + \Delta H_{22} \tag{B5}$$

Equations (B1) to (B5) can be solved simultaneously to give the following expression:

$$\Delta H_{11} = \frac{\dfrac{2g\ \Delta H_2}{C} + V_1 - V_2 - \dfrac{K}{A_D}(H_1 + \Delta H_1 - H_R)}{\dfrac{K}{A_D} + \dfrac{2g}{C}} \tag{B6}$$

The new conditions after wave action can then be computed from equations (B1) to (B5).

The change in accumulator pressure is computed by assuming that the line conditions adjacent to the accumulator do not change significantly over the short time interval. The compliance of the accumulator C' is assumed to be constant and is given as

$$C' = \frac{\Delta \mathcal{V}}{\Delta H_R} \tag{B7}$$

where $\Delta \mathcal{V}$ is the volume change of the propellant stored in the accumulator, and ΔH_R is the change in accumulator pressure. Over a short time interval Δt after wave action, the volume change can be given as

$$\Delta \mathcal{V} = Q_2\ \Delta t \tag{B8}$$

Therefore, the change in accumulator pressure over a short time interval following wave action is

$$\Delta H_R = \frac{Q_2\ \Delta t}{C'} \tag{B9}$$

Because of the short interval considered in the digital computer calculations, the change in accumulator pressure is computed without considering the change in line conditions that would naturally accompany this change. The new accumulator pressure is used in the subsequent set of calculations.

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